What are the units? That is a useful first question whenever you read a chart, calculate a quantity, or compare two numbers. A running total and a rate of change may come from the same data, but they answer different questions. A running total tells you how much has accumulated up to a particular time. A rate of change tells you how quickly a quantity is increasing or decreasing over an interval.

Confusing the two can lead to everyday mistakes: treating a shipment count as a shipping speed, interpreting monthly sales as a sales rate, or reading a cumulative balance instantaneous rate as the amount earned in one period. This guide explains the difference between a running total and a rate of change, how to calculate each, why time intervals matter, and how units and graphs help you interpret them correctly.

The short answer: two different questions

Suppose a warehouse ships boxes throughout the day. You could ask:

- **"How many boxes have shipped so far?"** This asks for a running total.
- **"How quickly are boxes shipping?"** This asks for a rate of change.

The first answer might be 240 boxes. The second might be 30 boxes per hour. Those answers are not interchangeable. The running total is an amount; the rate is an amount divided by time.



Idea Question it answers Example Typical units Running total How much has accumulated by this time? 240 boxes shipped by noon boxes Rate of change How quickly is the quantity changing? 30 boxes shipped per hour boxes per hour

What does "cumulative total" mean?

A **cumulative total** is an amount built by adding new contributions to what was already counted. "Cumulative" means that earlier amounts remain included. A **running total** is the cumulative amount reported as the data progresses, often at regular times such as each hour, day, or month.

For example, imagine a shop sells 12, 9, and 15 mugs on Monday, Tuesday, and Wednesday. The daily sales are 12, 9, and 15 mugs. The running totals are 12 mugs by Monday's end, 21 by Tuesday's end, and 36 by

Wednesday's end.

Day Mugs sold that day Running total sold Monday 12 mugs 12 mugs Tuesday 9 mugs 21 mugs Wednesday 15 mugs 36 mugs

The daily sales answer, "How many mugs were sold on that day?" The running total answers, "How many mugs have been sold altogether through that day?" Both quantities use units of mugs, but their meanings differ. A label such as "sales" is not enough by itself; the chart should make clear whether it shows sales per day or cumulative sales.

What does "rate of change" mean?

The **rate of change meaning** is how much a quantity changes compared with a change in another quantity—often time. If a value rises by 18 units over 3 hours, its average rate of change is 6 units per hour.

In general, for a quantity Q recorded at two times t_1 and t_2, the average rate of change is

$$\textaverage rate of change =\fracQ(t_2)-Q(t_1)t_2-t_1.$$

The numerator is the change in the recorded quantity. The denominator is the elapsed time. The units follow the same division: if Q is measured in boxes and time is measured in hours, the rate is in boxes per hour.

Suppose a running shipment total is 80 boxes at 9 a.m. and 140 boxes at 11 a.m. The clean question is: "What was the average number of boxes shipped per hour from 9 a.m. to 11 a.m.?" The total increased by 60 boxes over 2 hours, so

$$\frac140-80\text boxes11-9\text hours =\frac60\text boxes2\text hours =30\text boxes per hour.$$

The answer is an average over that two-hour interval. It does not tell us that exactly 30 boxes shipped in every hour—or that boxes shipped at a perfectly steady pace.

Why the interval matters

A rate is incomplete unless you know the interval used to calculate it. "The total increased by 60 boxes" describes a change in amount, but it is not a rate. "The total increased by 60 boxes in 2 hours" gives both the change and the interval, allowing us to calculate 30 boxes per hour.

Different intervals can produce different average rates. Suppose a tank's recorded water amount is 20 liters at 8 a.m., 50 liters at 9 a.m., and 80 liters at noon.

- From 8 to 9 a.m., the average rate is 30 liters per hour.
- From 9 a.m. to noon, the average rate is $30/3=10$ liters per hour.
- From 8 a.m. to noon, the average rate is $60/4=15$ liters per hour.

All three calculations use the same recorded quantity, but they answer questions about different time intervals. If a dashboard reports an average rate, check the start and end times before comparing it with another rate.

Be precise about hours and minutes

Units can also expose a time-conversion error. If 24 packages are processed in 30 minutes, the rate is 48 packages per hour, not 24 packages per hour. Since 30 minutes is half an hour, the clean calculation is $24\text packages/0.5\text hour=48\text packages per hour$. State the time unit instead of silently mixing minutes and hours.

Define what is recorded and when counting starts

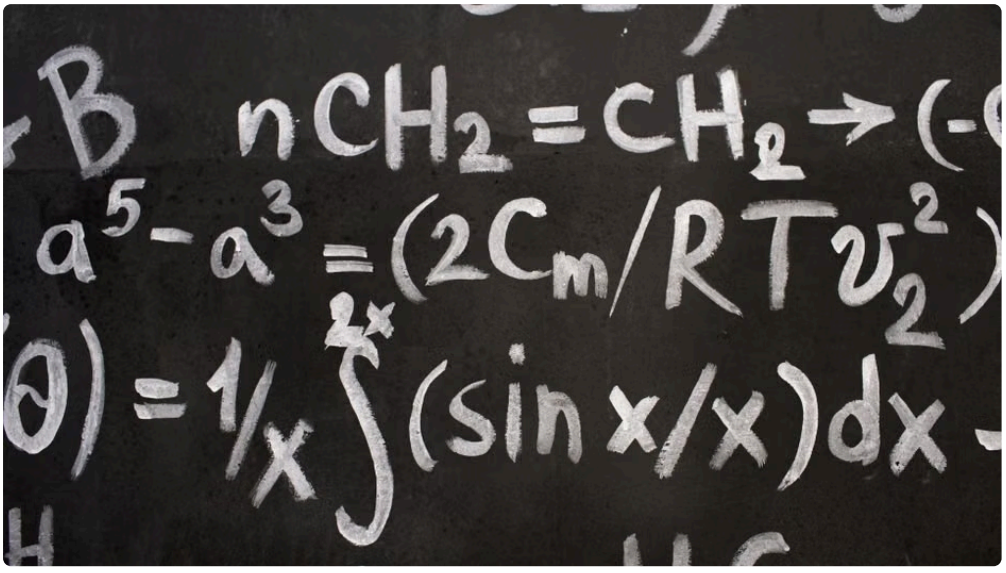
Before interpreting a running total or calculating its rate, identify exactly what the recorded quantity represents. Is it boxes dispatched, boxes received, dollars deposited, or orders completed? Next, identify the start time and what the total includes.

For instance, a chart might show “orders completed today.” Does the count reset to zero at midnight, at the start of a work shift, or at the beginning of the month? A total of 75 could be a daily total, a shift total, or a year-to-date total. Those are different quantities, even if the number is the same.

A helpful way to rewrite a word problem before calculating is to make one clean sentence: “Find the average change in the number of completed orders, in orders per hour, from 10 a.m. to noon.” That sentence names the quantity, units, and interval. It also prevents the common mistake of giving a total when the question asks for a rate—or a rate when the question asks for a total.

Reading totals and rates on graphs

On a graph of a running total versus time, the vertical value gives the accumulated amount at that time. The steepness of the graph describes how quickly the total is changing. A steep upward segment means the total is increasing quickly; a flatter segment means it is increasing more slowly. A downward segment means the recorded total is decreasing, as could happen with a bank balance after withdrawals or inventory after sales.



For a straight segment between two points, its slope is the average rate of change over that interval:

\[\textslope =\frac\textchange in vertical quantity\textchange in time. \]

But not every data display should connect points with straight lines. If a shipping system records updates only when a delivery is scanned, a step graph may be more appropriate: the total stays at one level between recorded events, then jumps when a new shipment is counted. Drawing diagonal lines between those timestamps can suggest that the shipment was counted gradually, even when the data do not say that.

For example, if inventory is recorded as 40 units at 9 a.m. and 65 units at 10 a.m. because a delivery was scanned at 10, the data may show a jump from 40 to 65 at that event. They do not necessarily show inventory rising steadily through the hour. Choose a graph style that matches how and when the data were collected, and label both axes with quantities and units.

Totals and rates in dashboards

Workplace dashboards often place totals and rates near each other. Reading the label carefully helps prevent errors:

- **"Units shipped today"** is usually a total, measured in units.
- **"Units shipped per hour"** is a rate, measured in units per hour.
- **"Revenue this month"** is a total amount of money, measured in dollars.
- **"Revenue per day"** is a rate, measured in dollars per day.
- **"Average order value"** is a ratio, usually dollars per order—not dollars per hour.

Not every rate uses time as its denominator. A rate compares quantities, and its units show which quantities are being compared. For example, miles per gallon measures distance per amount of fuel; dollars per item measures cost per item. Always check the denominator rather than assuming that every rate is "per hour."

A practical checklist

1. **Name the recorded quantity.** What is being counted or measured?
2. **Find the starting point.** When does the total begin, and does it reset?
3. **Read the question.** Does it ask "how much altogether?" or "how quickly does it change?"
4. **Identify the interval.** For a rate, specify the beginning and ending times or other comparison values.
5. **Write the units.** Check that the final units match the question and that time units are consistent.
6. **Match the graph to the data.** Use steps for jumps at events when a continuous change between observations is not supported.

Using MathJax and Runestone to practice

MathJax is a tool for displaying mathematical notation clearly in web pages. In a lesson that supports MathJax, the average-rate formula can appear as readable math rather than plain-text symbols. That is especially useful when students are comparing changes in totals over different intervals.

Runestone is an interactive publishing platform used for digital learning materials. A Runestone lesson can provide practice questions, immediate feedback, and opportunities to interpret charts. For this topic, useful practice would ask learners to distinguish a daily amount from a cumulative amount, calculate an average rate over a stated interval, and explain the units in a sentence.

The key idea

A running total tells you the accumulated amount up to a particular time; a rate of change tells you how much that amount changes per unit of another quantity, often time. The running total and the rate can be calculated from the same records, but they are different answers to different questions.

When you are unsure, pause and ask: What is being recorded? When does counting start? What interval is being considered? And, most importantly, what are the units? Those questions make the difference between a number that merely looks plausible and an answer that actually means what the chart or problem asks.